Statistical Estimation of Service Cracks and Maintenance Cost for Aircraft Structures

J.-N. Yang*
George Washington University, Washington, D.C.

A method is developed for the statistical estimation of the number of cracks to be repaired in service as well as the repair and maintenance costs. The present approach accounts for the statistical distribution of the initial crack size, the statistical uncertainty of the NDI technique used for detecting the crack, and the crack propagation of repaired (renewal) details. The mean and the standard deviation of the cumulative number of cracks to be repaired are computed. The statistics of the costs of repair and maintenance, expressed in terms of the percentage of the cost of replacement, are estimated as a function of service time. It is shown that the cost of repair or maintenance increases so rapidly after a certain service time that the replacement or retirement of the structure may be necessary. The results of the present study provide relevant information for the decision of fleet management, the estimation of life cycle cost and procurement specifications. Numerical results are worked out to demonstrate the significant effect of the inspection frequency on the repair and the maintenance costs.

Nomenclature

$a(0)$ = initial crack size
$a(T)$ = crack size at Tth flight
C_I = cost of inspecting one detail
C_k = cost of repairing one kth level crack
$C(j)$ = cumulative maintenance cost, including costs of inspection and repair, in the service interval $(0,jT)$
$\bar{C}(j)$ = mean (average) value of $C(j)$
$f_{a(0)}(x)$ = probability density of initial crack size $a(0)$ of nondefective population
$f_{a^*(0)}(x)$ = probability density of initial crack size $a^*(0)$ of defective population
$f_{a(T-)}(x)$ = probability density of crack size, $a(T-)$, at T before the first inspection
$F(y)$ = probability of detecting a crack size y during inspection in service
$F^*(y)$ = $1-F(y)$; probability of missing a crack size y during inspection in service
$I(j,k)$ = total cumulative number of kth level cracks detected and repaired in the service interval $(0,jT)$ in one region
$\bar{I}(j,k)$ = mean (average) value of $I(j,k)$
$L(y)$ = probability of detecting a crack size y during the initial inspection prior to service
$L^*(y)$ = $1-L(y)$ = probability of missing a crack size y during the initial inspection prior to service
$N(i,k)$ = no. of kth level cracks (in nondefective population) detected and repaired at the ith inspection
$N^*(i,k)$ = no. of kth level cracks (in defective population) detected and repaired at the ith inspection
$\bar{N}(i,k)$ = mean (average) value of $N(i,k)$
$\bar{N}^*(i,k)$ = mean (average) value of $N^*(i,k)$
N_1 = total number of details in one region associated with nondefective population
N_2 = total number of details in one region associated with defective population
$p_1(i,k)$ = probability of detecting (or repairing) a kth level crack (in nondefective population) at the ith inspection
$p_2^*(i,k)$ = probability of detecting a kth level crack at the ith inspection, contributed by unrepaired detail (associated with defective population) population)
$p_1(i)$ = probability of detecting (or repairing) a crack of any size (in nondefective population) at ith inspection
$p_2(i,k)$ = probability of detecting a kth level crack (in defective population) at the ith inspection
$p_2^*(i,k)$ = probability of detecting a kth level crack at the ith inspection, contributed by unrepaired detail (associated with defective population)
$p_2(i)$ = probability of detecting a crack of any size (in defective population) at ith inspection
Q = constant depending on material properties and flight loads, Eq. (10)
$\sigma_C^2(j)$ = variance of $C(j)$
$\sigma_I^2(j,k)$ = variance of $I(j,k)$
$\sigma_N^2(i,k)$ = variance of $N(i,k)$
$\sigma_{N^*}^2(i,k)$ = variance of $N^*(i,k)$
$\eta,\gamma,\epsilon,\lambda$ = parameters associated with Johnson S_u distribution, Eq. (28)

$\mathcal{I}(j,k)$, $\bar{\mathcal{I}}(j,k)$, $\sigma_{\mathcal{I}}^2(j,k)$, $\mathcal{C}(j)$, $\bar{\mathcal{C}}(j)$, $\sigma_{\mathcal{C}}^2(j)$ are, respectively, identical to $I(j,k)$, $\bar{I}(j,k)$, $\sigma_I^2(j,k)$, $C(j)$, $\bar{C}(j)$, $\bar{C}(j)$, $\sigma_C^2(j)$, except that the former are quantities for the entire component while the latter are quantities for one single region in the component.

I. Introduction

THE cost of maintenance, including the costs of inspection, repair, and replacement, for a fleet of aircraft, in order to maintain a certain level of fleet reliability, is of significantly practical importance. Theoretically, the service life of an airplane can be extended for a very long time as long as inspections, repairs, and replacements are performed frequently enough. However, it will become apparent that after a certain service time called economical life, the maintenance cost increases so rapidly that it may no longer be worthwhile to maintain the aircraft and it should be retired. From the procurement standpoint, the ratio of the maintenance cost to the initial cost of aircraft is a useful informative in establishing the contractual specifications for procurement requirements.

Received May 22, 1975; presented as Paper 75-767 at the AIAA/ASME/SAE 16th Structures, Structural Dynamics, and Materials Conference, Denver, Colo., May 27-29, 1975; revision received April 7, 1976. This research is supported by NASA Langley Research Center through Grant NSG 1099.

Index category: Reliability, Quality Control, and Maintainability.

*Associate Professor, Department of Civil, Mechanical, and Environmental Engineering. Member AIAA.

In the cost optimization for inspection maintenance procedures as presented in Ref. 1 the objective function to be minimized is the total statistically expected cost that consists of the expected cost of failure and the expected cost of inspection and maintenance. The expected cost of failure is the cost of failure multiplied by the probability of structural failure. While the methodology to estimate the probability of structural failure under periodic inspections has been available,[1-6] the technique for the statistical estimation of maintenance cost has not been formulated.

It is the purpose of this paper to present a theoretical formulation and computational procudure for the statistical estimation of the number of cracks to be repaired, the repair cost, and maintenance cost as a function of service time. As such, this study will provide information for use in the overall optimization of maintenance procedures as discussed in Ref. 1. The results of this study may also serve as a rational basis for the fleet management to lower maintenance costs, the estimation of life cycle cost and the specification of procurement requirements, e.g., the statistics of the cost of repair should be within certain allowable bounds.

A simple crack propagation law[7-13] is applied to predict the statistical distribution of crack size under service loads. A periodic inspection[1-6] is performed to detect the cracks. The detection of an existing crack of size y using a NDI technique involves statistical uncertainties,[1-6,14] and it is considered as a random variable. When a cracked detail is detected during inspection, it is repaired. The repaired details, referred to as *renewal details,* will proceed to develop into cracks of larger size and be detected and repaired again after a longer service time. The crack propagation of repaired (renewal) details is taken into account in the present study.

A formulation for estimating the mean value and the standard deviation of the cumulative number of cracked details to be repaired is established as a function of service time. The mathematical technique employed here is a successive transformation of a random variable denoting the crack size. The formulation takes into account a) the statistical distribution of initial crack size, b) the statistical uncertainty of NDI techniques for crack detection, and c) the crack propagation of repaired (renewal) details.

The cost of repair depends not only on the number of detected cracked details, but also on the size of the crack. Hence, effort is first made to estimate the average (mean) value and the standard deviation of the cumulative number of cracks with sizes in different ranges. Then, the average (mean) value and the standard deviation of the cumulative cost of maintenance, including the costs of inspection and repair, are estimated as functions of service time. Finally, numerical examples are worked out to demonstrate the significant effect of the inspection frequency on the statistics of service cracks and repair costs.

II. Preliminary

Prior to service, two populations of initial details (items) are considered, referred to as the defective population and the nondefective population, respectively. The defective population consists of initial cracks of detectable size. These cracks are either inherent in materials or introduced during a fabrication process and assembling of structures. They propagate and develop into critical cracks during early service time (see Fig. 1). Past experiences[15] indicate that failure caused by such existing cracks accounts for a considerable percentage of the overall aircraft failures.

Initial cracks in the nondefective population are not detectable by the NDI techniques prior to service. However, they can be detected and repaired later when they develop into detectable crack sizes as shown in Fig. 1. Recently, the statistical distribution of the initial crack size in this population has been available for some types of aircraft. Such a statistical distribution is obtained from the results of laboratory tests. When detectable cracks are observed during the test, they are extrapolated backwardly by the use of crack propagation laws to estimate their initial crack sizes (e.g., Refs. 16-17) as shown in Fig. 1.

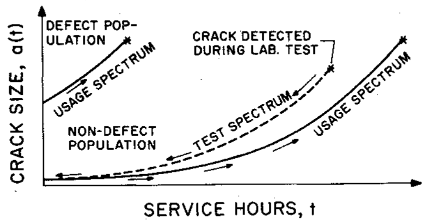

Fig. 1 Crack population and propagation.

The reliability of aircraft structures is significantly influenced by the inspection frequency or the inspection interval.[1-6] In this study, the periodic inspection interval, T flights, is a given parameter, which ensures a specified level of aircraft reliability.[1-6]

The cost of repairing one crack depends on the size of the crack. For the crack size between 0.01-0.03 in., it is only necessary to ream the hole to the next size and the cost of repair is least expensive. Cracks in this size range are referred to as first-level cracks and the associated repair cost as the cost for first-level repair. For cracks with sizes between 0.03-0.1 in., a regular retrofit maintenance may be required. These are referred to as the second-level cracks. The cost for the second-level repair is, of course, higher than the cost for the first-level repair. For the sake of simplicity in presentation, 4 levels of repair (i.e., 4 levels of cracks) are assumed to be sufficient to cover all possible crack sizes greater than 0.01 in. It should be emphasized that the formulation presented can take into account n (>4) levels of cracks (n levels of repair).

III. Formulation

Supposing that a major aircraft structural component, such as a wing, fuselage, etc., is divided into M regions, the stress level at each location of a region is approximately the same while it varies from one region to another. An airplane is subjected to a periodic inspection at $t_j = jT$, $j = 0, 1, 2, \ldots$, where T is the inspection interval, and $j = 0$ is the initial inspection prior to service.

In one region, let $N(i,k)$ be the number of kth level cracks (cracked items) detected and repaired at the ith inspection performed at $t_i = iT$, associated with the nondefective population, and $N^*(i,k)$ associated with the defective population. Both $N(i,k)$ and $N^*(i,k)$ are random variables. Then, the total cumulative number of kth level cracks repaired in the service time interval $(0, jT)$, denoted by $I(j,k)$, is

$$I(j,k) = \sum_{i=0}^{j} [N(i,k) + N^*(i,k)] \tag{1}$$

With the approximation that the event of cracking for each individual item (detail) is statistically independent, the statistical distributions of $N(i,k)$ and $N^*(i,k)$ follow the binomial distribution

$$P[N(i,k) = n] = \binom{N_1}{n} p_1^n(i,k) [1 - p_1(i,k)]^{N_1 - n}$$

$$P[N^*(i,k) = n] = \binom{N_1}{n} p_2^n(i,k) [1 - p_2(i,k)]^{N_2 - n} \tag{2}$$

in which N_1 and N_2 are the total number of items (or details) in one region, associated with the nondefective and the defective populations, respectively; $p_1(i,k)$ and $p_2(i,k)$ are the probabilities of detecting (or repairing) a kth level crack (item) at the ith inspection, respectively, associated with the nondefective and the defective populations. The approximation of statistically independent cracking for each detail has been made due to the lack of data on the correlation of cracking and due to the close relationship between the fatigue life and cracking.

The mean values, $\bar{N}(i,k)$ and $\bar{N}^*(i,k)$, and the variances, $\sigma_N^2(i,k)$ and $\sigma_{N^*}^2(i,k)$, of both $N(i,k)$ and $N^*(i,k)$ are, respectively, as follows:

$$\bar{N}(i,k) = N_1 p_1(i,k)$$

$$\sigma_N^2(i,k) = N_1 p_1(i,k)[1-p_1(i,k)]$$

$$\bar{N}^*(i,k) = N_2 p_2(i,k)$$

$$\sigma_{N^*}^2(i,k) = N_2 p_2(i,k)[1-p_2(i,k)] \tag{3}$$

As a result, the mean value, $\bar{I}(j,k)$, and the variance, $\sigma_I^2(j,k)$, of the total cumulative number of kth level cracks repaired in the service time interval $(0,jT)$ can be obtained from Eqs. (1) and (3) as follows:

$$\bar{I}(j,k) = \sum_{i=0}^{j} [\bar{N}(i,k) + \bar{N}^*(i,k)]$$

$$\sigma_I^2(j,k) = \sum_{i=0}^{j} [\sigma_N^2(i,k) + \sigma_{N^*}^2(i,k)] \tag{4}$$

in which $\bar{N}(i,k)$, $\bar{N}^*(i,k)$, $\sigma_N^2(i,k)$, and $\sigma_{N^*}^2(i,k)$ are given by Eqs. (3).

Let C_I and C_k be the cost of inspecting one item (detail) and the cost of repairing one kth level crack, respectively. Then, the total cumulative cost of maintenance, including the costs of repair and inspection, within the service time interval $(0,jT)$ can be written as

$$C(j) = \sum_{k=1}^{4} C_k I(j,k) + (N_1 + N_2) j C_I \tag{5}$$

in which the first term denotes the total cumulative repair cost and the second term denotes the total cumulative inspection cost in $(0,jT)$.

The mean, $\bar{C}(j)$, and the variance, $\sigma_c^2(j)$, of the total cumulative cost of maintenance, $C(j)$, can be obtained from Eqs. (4) and (5),

$$\bar{C}(j) = \sum_{k=1}^{4} \bar{I}(j,k) C_k + jC_I(N_1 + N_2)$$

$$\sigma_C^2(j) = \sum_{k=1}^{4} \sigma_I^2(j,k) C_k^2 \tag{6}$$

in which $\bar{I}(j,k)$ and $\sigma_I^2(j,k)$ are given by Eqs. (4).

The means and the variances of both the cumulative number of cracks repaired in $(0,jT)$ and the cumulative maintenance cost in $(0,jT)$ given respectively, by Eqs. (4) and (6), are valid for a particular region, say the mth region. For the entire structural component, e.g., wing or fuselage, the cumulative number of kth level cracks repaired in $(0,jT)$, denoted by $\boldsymbol{I}(j,k)$, and the cumulative maintenance cost in $(0,jT)$, denoted by $\boldsymbol{C}(j)$, are respectively,

$$\boldsymbol{I}(j,k) = \Sigma\, I(j,k); \quad \boldsymbol{C}(j,k) = \Sigma\, C(j,k) \tag{7}$$

in which the summation is taken over all regions of the entire component. The mean values and the variances of $\boldsymbol{I}(j,k)$ and $\boldsymbol{C}(j)$, can be obtained, respectively, from Eqs. (7, 6, and 4) as follows:

$$\boldsymbol{I}(j,k) = \Sigma\, \bar{I}(j,k); \quad \sigma_I^2(j,k) = \Sigma\, \sigma_I^2(j,k) \tag{8}$$

$$\bar{\boldsymbol{C}}(j) = \Sigma\, \bar{C}(j); \quad \sigma_C^2(j) = \Sigma\, \sigma_C^2(j) \tag{9}$$

It follows from Eqs. (1–9) that the means and standard deviations of both the cumulative number, $\boldsymbol{I}(j,k)$, of kth level cracks repaired in $(0,jT)$ and the cumulative maintenance cost in the service interval $(0,jT)$, $\boldsymbol{C}(j)$, can be determined, once $p_1(i,k)$ and $p_2(i,k)$ are computed. Methods for evaluating both quantities will be discussed next. It is mentioned again that $p_1(i,k)$ and $p_2(i,k)$ are the probabilities of detecting a kth level crack during the ith inspection associated with the defective and the nondefective populations, respectively.

IV. Derivation for $p_1(i,k)$ and $p_2(i,k)$

A. Crack Propagation under Service Loads

Under flight-by-flight service loads, a simple crack propagation law is used[1-5,7-11]

$$\mathrm{d}a(t)/\mathrm{d}t = Qa^b(t) \tag{10}$$

in which t = number of flights, $a(t)$ = crack size at t flights, b = material constant, Q = parameter depending on materials/structure properties and flight loads.

For the sake of simplicity in presentation, b is assumed to be 2.0, a typical value for aluminum materials. When b is different from 2, the following approach can be modified without difficulty. Integrating Eq. (10) from $a(0)$ (initial crack size) to $a(T)$ (the crack size at T), one obtains

$$a(T) = a(0)/[1 - QTa(0)] \tag{11}$$

in which Q is obtained from laboratory test results under simulated flight-by-flight service loads.[7-10] If test results are not available, it can be estimated using the method of cycle-by-cycle count, i.e., by the analysis of stress range spectra and material properties, such the crack propagation parameters.[3-7,11-13] In such a case, the load sequence effects (e.g., retardation) should be taken care of appropriately.[13]

The crack size at the ith inspection, $a(iT)$, can be expressed in terms of the crack size at the previous inspection, $a[(i-1)T]$, by the integration of Eq. (10) as

$$a(iT) = a[(i-1)T]/\{1 - QTa[(i-1)T]\} \tag{12}$$

Both Eqs. (11) and (12) will be used later for the transformation of random variables.

B. Nondefective Population

Let $f_{a(0)}(x)$ be the probability density function of the initial crack size, $a(0)$, for the nondefective population. Such a statistical distribution of initial crack sizes is available for some types of aircraft (see Fig. 2).

The probability density function $f_{a(T-)}(y)$ of the crack size $a(T-)$ right before the first inspection can be obtained from $f_{a(0)}(y)$ through the transformation of Eq. (11),[18]

$$f_{a(T-)}(y) = f_{a(0)}[y/(1+QTy)]\,|J(y)| \tag{13}$$

in which $|J(y)|$ is the absolute value of $J(y)$ and it follows from Eq. (11) that

$$J(y) = \mathrm{d}a(0)/\mathrm{d}a(T-)\Big|_{a(T-)=y} = 1/[1+QTy]^2 \tag{14}$$

The probability of detecting a crack of size y during inspection, denoted by $F(y)$, depends on the particular NDI technique and the crack size y. In general, $F(y)$ is an increasing function of y with $0 \le F(y) \le 1$. Empirical results for $F(y)$ are available for various NDI techniques.[1-6,14]

The probability of detecting a kth level crack at the *initial inspection* prior to service, denoted by $p_1(0,k)$, is zero for nondefective population. The probability $p_1(1,k)$ of detect-

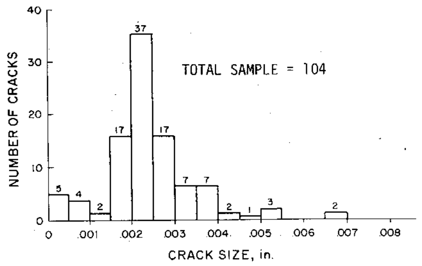

Fig. 2 Distribution of initial crack size (nondefective population).

ing a kth level crack at the first inpection is

$$p_I(I,k) = \int_{a_{k-I}}^{a_k} F(y) f_{a(T-)}(y)\mathrm{d}y$$

$$= \int_{a_{k-I}}^{a_k} F(y) f_{a(0)}\left(\frac{y}{I+QTy}\right) |J(y)| \mathrm{d}y \tag{15}$$

in which $F(y)f_{a(T-)}(y)\mathrm{d}y$ is the probability of detecting a crack whose size is in the interval $(y,y+\mathrm{d}y)$. The crack with size in the range a_{k-I} to a_k is referred to as the kth level crack, and in the present discussion, $k=I,2,3,$ and 4.

Let $p_I^*(i,k)$ be the *unconditional* probability of detecting a kth level crack *belonging to the unrepaired detail* (item has not been repaired before) during the ith inspection, when a detail (may be repaired or unrepaired) is under inspection. In other words, $p_I^*(i,k)$ is the contribution from the unrepaired details. Then $p_I^*(i,k)$ can be derived by use of the successive transformation of Eq. (12) (see Appendix for derivation) as follows:

$$p_I^*(i,k) = \int_{a_{k-I}}^{a_k} F(y_I)\left[\prod_{n=2}^{i} F^*(y_n)\right] f_{a(0)}(y_{i+I})$$

$$\times\left[\prod_{n=I}^{i} |J(y_n)|\right]\mathrm{d}y_I$$

$$y_{n+I} = y_n/[I+TQy_n] = y_I[I+nTQy_I] \tag{16}$$

in which

$$\prod_{n=2}^{i} F^*(y_n)$$

should be replaced by one for $i=I$, and $F^*(y) = I - F(y)$ is the probability of not detecting (or missing) a crack of size y during the inspection.

It is important to note that $p_I^*(i,k)$ is *not* the *conditional* probability of detecting a kth level crack at the ith inspection under the condition that an unrepaired crack is inspected. As the service time increases, the number (or percentage) of unrepaired details decreases resulting from the repair of detected cracks. Since $p_I^*(i,k)$ is an *unconditional* probability contributed by unrepaired details, the decrease in the number of unrepaired details has been accounted for in the derivation of $p_I^*(i,k)$ given by the Appendix.

Since there are no details repaired at the initial inspection, it is obvious from Eqs. (15) and (16) that

$$p_I^*(I,k) = p_I(I,k) \tag{17}$$

The repaired (or renewal) details may develop into detectable crack size and be detected (and repaired) again during later service time. The crack propagation of repaired detail can be accounted for in the following manner:

Let $p_I(i)$ be the probability of repairing (detecting) a crack of any size (in any level) at the ith inspection time, i.e.,

$$p_I(i) = \sum_{k=I}^{4} p_I(i,k) \tag{18}$$

Then the probability of detecting (or repairing) a kth level crack at the second inspection, denoted by $p_I(2,k)$, is

$$p_I(2,k) = p_I^*(2,k) + p_I(I)p_I^*(I,k) \tag{19}$$

in which $p_I^*(2,k)$ is the contribution from the unrepaired details, and $p_I(I)p_I^*(I,k)$ is the contribution from details repaired (or renewed) at the first inspection time T.

At the third inspection, the probability of detecting a kth level crack, attributed to the details repaired (renewed) at the first inspection but not repaired at the second inspection is $p_I(I)p_I^*(2,k)$, and hence

$$p_I(3,k) = p_I^*(3,k) + p_I(I)p_I^*(2,k) + p_I(2)p_I^*(I,k) \tag{20}$$

in which $p_I^*(3,k)$ is attributed to unrepaired details and $p_I(2)p_I^*(I,k)$ is attributed to the details repaired (renewed) at the second inspection.

In a similar manner, the general solution for the probability of detecting a kth level crack at ith inspection, $p_I(i,k)$, can be derived as

$$p_I(i,k) = p_I^*(i,k) + \sum_{n=I}^{i-I} p_I(n)p_I^*(i-n,k) \tag{21}$$

for $i=2,3,...;$ $k=I,2,3,4$; and in which $p_I^*(i,k)$ is given by Eqs. (16) and $p_I(n)$ is given by Eq. (18). The solution for $p_I(I,k)$ is given by Eqs. (15). Hence, Eqs. (15, 16, 18, and 21) constitute a formal recurrence solution for $p_I(i,k)$.

C. Defective Population

Let $f_{a^*(0)}(x)$ be the probability density function of the initial crack size for the defective population. An initial inspection is performed at $t=0$ prior to service. Let $L(y)$ and $L^*(y)$ denote the probabilities of detecting and not detecting (missing) a crack of size y, respectively, during the initial inspection, i.e., $L^*(y) = I - L(y)$. The initial inspection may be more stringent than the field inspection and hence $L(y)$ is not equal to $F(y)$.

The probability $p_2(0,k)$ of detecting a kth level crack during the initial inspection is

$$p_2(0,k) = \int_{a_{k-I}}^{a_k} L(y) f_{a^*(0)}(y)\mathrm{d}y \tag{22}$$

and the probability, $p_2(0)$, of detecting a crack of any size (in any level) at the initial inspection is

$$p_2(0) = \sum_{k=I}^{4} p_2(0,k) \tag{23}$$

Let $p_2^*(i,k)$ be the *unconditional* probability of detecting a kth level crack belonging to the unrepaired details at the ith inspection, when a detail (may be repaired or unrepaired) is inspected. Then, in a similar manner as for the nondefective population [see Eq. (16)], one obtains

$$p_2^*(i,k) = \int_{a_{k-I}}^{a_k} L^*(y_I)F(y_I)\left[\prod_{n=2}^{i} F^*(y_n)\right]$$

$$\times f_{a^*(0)}(y_{i+I})\left[\prod_{n=I}^{i} |J(y_n)|\right]\mathrm{d}y_I \tag{24}$$

$$y_{n+I} = y_n/[I+QTy_n] = y_I/[I+nTQy_I] \tag{25}$$

in which the term

$$\prod_{n=2}^{i} F^*(y_n)$$

should be replaced by one for $i=I$.

It is assumed that the statistical distribution of the initial crack size for the repaired details is the same as that of the initial crack size for the nondefective population. As a result, after a detail is repaired, its initial crack size will have a density function $f_{a(0)}(x)$. Hence, at the first inspection, the probability of detecting a kth level crack contributed by the details repaired (renewed) at the initial inspection is $p_2(0)p_1^*(1,k)$. Consequently,

$$p_2(1,k) = p_2^*(1,k) + p_2(0)p_1^*(1,k) \tag{26}$$

in which $p_2^*(1,k)$, $p_2(0)$, and $p_1^*(1,k)$ are given by Eqs. (24, 23, and 16), respectively.

The probability of detecting a kth level crack, $p_2(i,k)$, at the ith inspection time can be derived in a similar manner; with the results

$$p_2(i,k) = p_2^*(i,k) + \sum_{n=0}^{i-1} p_2(n)p_1^*(i-n,k) \ ;$$

$$p_2(n) = \sum_{k=1}^{4} p_2(n,k) \ ; \qquad i=1,2,... \tag{27}$$

in which $p_2^*(i,k)$ and $p_1^*(i-n,k)$ are given by Eqs. (24) and (16), respectively. Equations (27) are the formal recurrence solution for $p_2(i,k)$, where $p_2(0,k)$ is given by Eq. (22).

V. Numerical Example and Discussion

A critical component of an aircraft with 100 details that are susceptible to crack propagation is considered. It is estimated that the stress level at the location of each detail is approximately the same and hence the component is considered as one region, i.e., $M=1$ in Eqs. (7). Approximately 98% of these details belongs to the nondefective population and the probability density of the initial crack size given by Fig. 2 is fitted reasonably well by a Johnson S_u distribution[18]

$$f_{a(0)}(y) = \frac{\eta}{\sqrt{2\pi}} \frac{1}{\sqrt{\lambda^2+(y-\epsilon)^2}} \exp\left\{-\frac{1}{2}\right.$$

$$\left.\times\left[\gamma+\eta \ln\left\{\bar{y}+(\bar{y}^2)^{1/2}\right\}\right]^2\right\}$$

$$\bar{y} = (y-\epsilon)/\lambda \tag{28}$$

in which η, λ, ϵ, and γ are four parameters determined from the first four central moments of the data given in Fig. 2.[18] These values are $\eta=1.672$, $\gamma=0$, $\epsilon=0.002481$ in., $\lambda=0.001616$ in. The Johnson distribution, which accounts for the first four central moments, is a more accurate representation than the conventional two-parameter distributions, e.g., normal, lognormal, etc.

The remaining 2% of details belongs to the defective population with an exponential distribution for the initial crack size[4]; $f_{a^*(0)}(y) = b \exp[-b(y-a_0)]$ for $y>a_0$ and $f_{a(0)}(y)=0$ for $y<a_0$, where $a_0=0.01$ in. and $b=30$ in. have been used.

For a specific service loading condition, Q appearing in Eq. (10) is 0.015 per inch per flight. It takes approximately 2200 flights for a crack of size 0.03 in. to grow to a critical crack size of 1 in., and it takes approximately 4400 flights for a crack of size 0.01 in. to grow to a size of 0.03 in. Hence, according to the current U.S. Air Force specification, a periodic inspection at a time interval $T=\min.\ (2200/2;4400)=1100$ flight is performed. The reliability analysis of aircraft structures under a given inspection frequency has been discussed in Refs. 1-6.

The probability of detecting a crack of size y, at the initial inspection depends on a particular NDI technique employed, and is approximated by an exponential function[4]; $L(y) = 1-\exp[-g(y-a_0)]$ for $y \geq a_0$ and $L(y)=0$ for $y<a_0$, where $g=20$ per inch is used. The probability of detecting a crack of size y, at the subsequent inspection is approximated by $F(y) = 1-\exp[-h(y-a_0)]$ for $y \geq a_0$ and $F(y)=0$ for $y<a_0$ where $h=15$ per inch for a particular NDI technique. It should be mentioned that the detection probabilities $L(y)$ and $F(y)$ increase as the crack size y increases.

The crack with size in the range 0.01-0.03 in. is referred to as the first level crack, 0.03-0.1 in. as the second level crack, 0.1-0.5 in. as the third level crack, and 0.5 in. or larger as the fourth level crack. Hence, in Eqs. (15, 16, 22, and 24), $a_0=0.01$ in., $a_1=0.03$ in., $a_2=0.1$ in., $a_3=0.5$, in., $a_4=\infty$.

The cost of repairing cracks of different levels is expressed as the percentage of the cost of replacing the entire component. In this particular example, $C_1=0.05\%$, $C_2=0.2\%$, $C_3=0.7\%$, and $C_4=2\%$ [Eqs. (5) and (6)]. The cost of inspecting one detail C_I is 0.005% of the replacement cost [Eqs. (5) and (6)]. One lifetime of the aircraft is 6600 flights and hence 6 inspections are performed during one lifetime.

With all the input information given above, the mean (average) of the cumulative number of repaired cracks (details), $\bar{I}(j,k)$, is computed as a function of service hours $jT(j=1,2,...)$ for four levels of cracks $(k=1,2,3,4)$ with the use of Eqs. (4, 3, 21, and 27). The results are plotted in Fig. 3a. The standard deviation, $\sigma_I(j,k)$, of cumulative number of repaired cracks is computed from Eqs. (3, 4, 21, and 27). The results are plotted in Fig. 3b as functions of service hours.

It is observed from Fig. 3a that the average cumulative number of repaired cracks increases very rapidly after the aircraft is in service for 2 lifetimes (12 inspections), and hence it may be more economical to retire the airplane. It is further observed that under the inspection interval of $T=1100$ flights, the number of repaired cracks (details), respectively, of the first level and the second level is very close. However, the number of repaired cracks of the third level or the fourth level is much smaller. As will be seen later, the number of repaired cracks in each level depends essentially on the inspection frequency.

The coefficient of variation, defined as $V(j,k)=\sigma_I(j,k)/\bar{I}(j,k)$, is a measure of the dispersion or the degree of uncertainty associated with the cumulative number of cracks to be repaired. It is observed from Fig. 3 that $V(j,k)$ is rather large within the first lifetime, indicating considerable uncertainty involved with regard to the number of cracks to be repaired. However, as the service time increases, the mean, $\bar{I}(j,k)$, increases much faster than the standard deviation, $\sigma_I(j,k)$, and hence the dispersion (or uncertainty) decreases significantly with respect to the increases of service hours. This is consistent with the observed fact that the longer the aircraft is in service, the higher the probability that most of the potential cracks will propagate to a detectable crack size and will be detected and repaired.

The mean of the cumulative repair cost [Eq. (16)]

$$\sum_{k=1}^{4} \bar{I}(j,k)C_k$$

is computed in terms of the percentage of the cost of replacement. The result is plotted as curve b in Fig. 4. The cumulative inspection cost $jC_I(N_1+N_2)$ [Eq. (6)] is plotted as curve c. Since the cost of inspection depends on the number of inspections only, it is a linear function of service hours, i.e., a straight line.

The mean (average), $\bar{C}(j)$, and the standard deviation, $\sigma_C(j)$, of the cumulative maintenance cost (including both the repair cost and the inspection cost) are computed from Eq. (6) in terms of the percentage of the cost of replacement. The results are plotted in Fig. 4 as curve a and curve d, respectively. As can be observed from Fig. 4, the cost of repair or maintenance increases very rapidly after two lifetimes. Furthermore, the cost of repair or maintentance involves considerable statistical dispersion (uncertainty) during early ser-

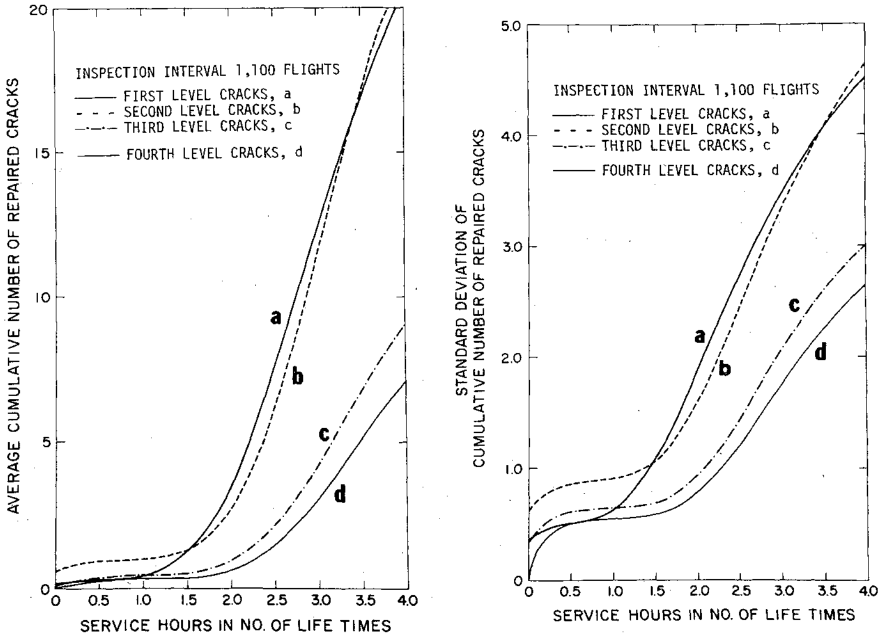

Fig. 3 Statistics of cumulative number of repaired cracks vs service hours; inspection interval $T = 1100$ flights; a) mean (average), b) standard deviation.

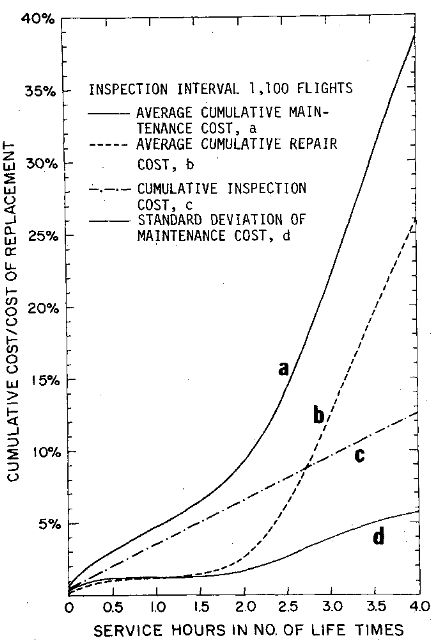

Fig. 4 Statistics of cumulative cost vs service hours; inspection interval $T = 1100$ flights.

vice time and the dispersion (uncertainty) diminishes as the service time increases.

The cost of maintenance (curve a) consists of the cost of inspection (curve c) and the cost of repair (curve b). It can be observed from Fig. 4 that within two lifetimes the cost of maintenance is essentially attributed to the cost of inspection since the cost of repair is very small. After two lifetimes, however, the cost of repair increases drastically because the cracks in the nondefective population have grown to detectable crack sizes, and hence the cost of repair becomes predominant as compared to the cost of inspection. As a result, for the particular set of input parameter values considered in this numerical example, the cost of mainenance is approximately proportional to the cost of inspection if the airplane is designed for two lifetimes.

The effect of inspection frequency on the repair and the maintenance costs is important in cost optimization for establishing an optimal maintenance policy.[1] When the inspection frequency increases, the cost of inspection increases accordingly, but the cost of repair may decrease because most cracks will be detected before growing into large sizes and hence fewer large cracks will be repaired. Although the inspection cost decreases as the inspection frequency decreases, the cost of repair may increase as a result of repairing more large cracks.

Let the total number of inspections in four lifetimes be 12, i.e., $N = 12$. In other words, the inspection interval T is 2200 flights. The means of the cumulative number of repaired cracks in different levels are plotted in Fig. 5a. The standard deviations are shown in Fig. 5b. It is observed from Fig. 5 that more cracks in the fourth level are expected to be repaired than any other level indicating that more large cracks are to be repaired.

The cumulative inspection cost, the average cumulative repair cost, the average cumulative maintenance cost, and the standard deviation of the cumulative maintenance cost are plotted in Fig. 6 as curve c, curve b, curve a, and curve d, respectively. It can be observed that the repair cost increases significantly over that in Fig. 4 because of the fact that more large cracks need to be repaired. Although the inspection cost decreases, the overall maintenance cost increases significantly due to a drastic increase in repair cost. It is observed that within two lifetimes the cost of repair and the cost of inspection are of equal magnitude. After two lifetimes, however, the cost of repair increases rapidly so that most of the maintenance cost (curve a) is attributed to the cost of repair (curve b).

Supposing that the inspection frequency is increased to 48, i.e., $N = 48$, the inspection interval T becomes 550 flights. The same statistical quantities as those given by Figs. 3-6 are displayed in Figs. 7 and 8. It can be observed from Fig. 7 that a large portion of repaired cracks is of the first level type (small cracks) and very few large cracks (fourth level cracks) are expected to be repaired. This is so because the inspection is performed frequently enough to detect the cracks before they develop into large sizes. It is observed from Fig. 8 that the repair cost is insignificant as compared to the cost of inspection. Hence, the cost of maintenance is essentially attributed to the cost of inspection, when the inspection frequency is high.

VI. Conclusions

A method has been developed for the statistical estimation of the number of service cracks, repair cost, and maintenance

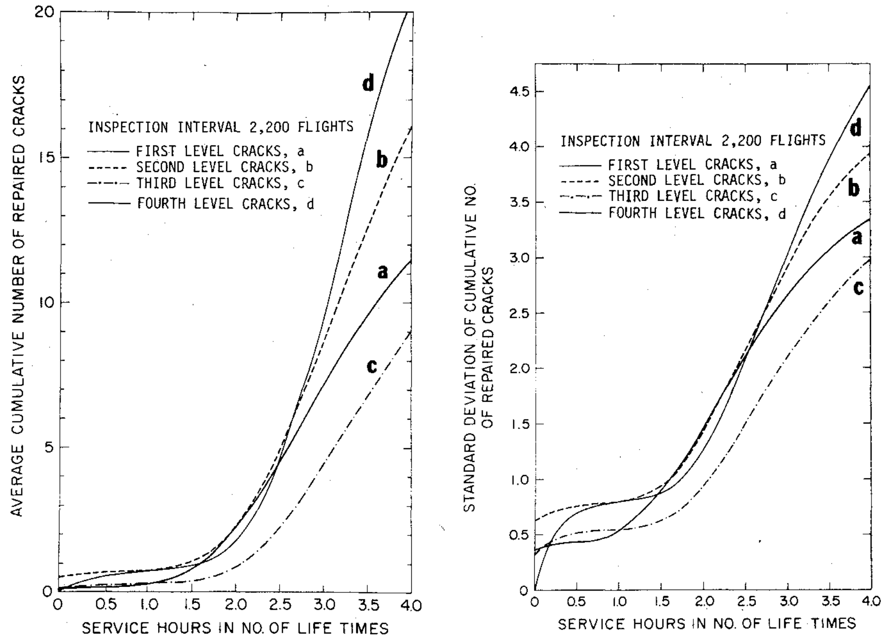

Fig. 5 Statistics of cumulative number of repaired cracks vs service hours; inspection interval $T = 2200$ flights; a) mean (average), b) standard deviation.

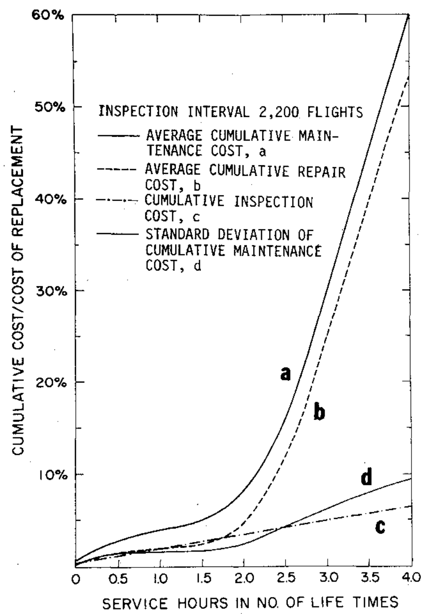

Fig. 6 Statistics of cumulative cost vs service hours; inspection interval $T = 2200$ flights.

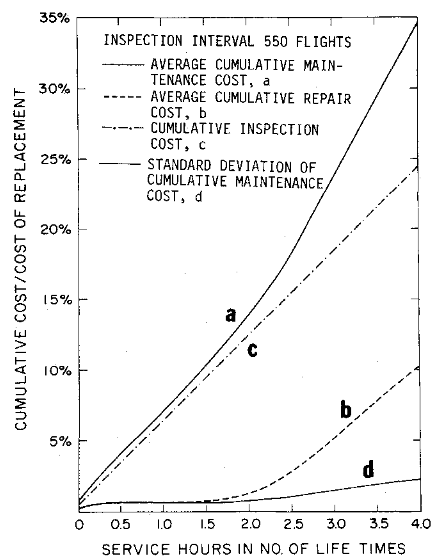

Fig. 8 Statistics of cumulative costs vs service hours; inspection interval $T = 500$ flights.

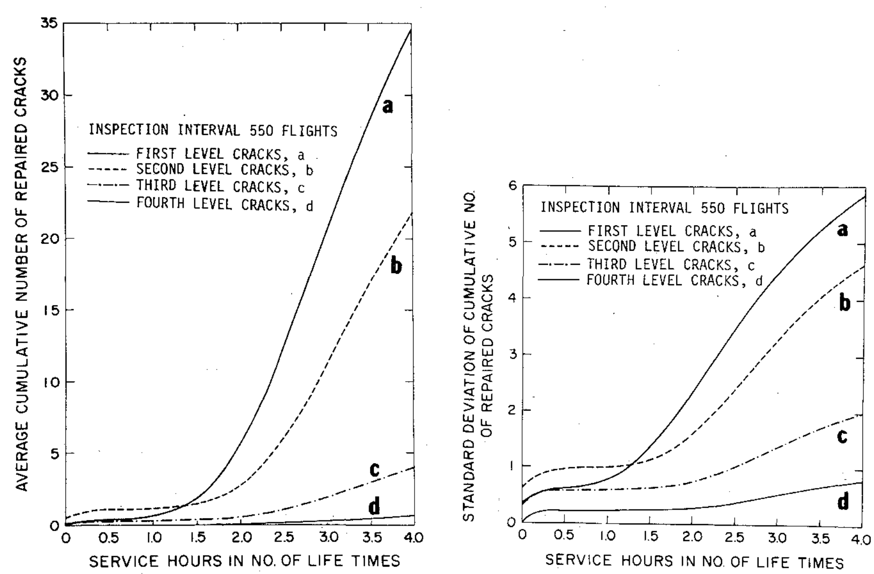

Fig. 7 Statistics of cumulative number of repaired cracks vs service hours; inspection interval $T = 550$ flights; a) mean (average), b) standard deviation.

cost. The present approach accounts for a) the statistical distribution of the initial crack size, b) the statistical uncertainty of the NDI technique used for detecting the crack, and c) the crack propagation of unrepaired and repaired details. It is demonstrated that the number of large cracks to be repaired decreases as the inspection frequency increases. When the inspection frequency is high, the cost of maintenance is essentially attributed to the cost of inspection, while the cost of maintenance is predominated by the cost of repair if the inspection frequency is low. It is further shown that when the inspection frequency is too low, not only the cost of maintenance is high but also the probability of structural failure is high,[2] thus resulting in a high expected cost of failure.

A simple crack propagation equation, Eq. (10), has been used[3,4,5,7-11,17] expediently in order to simplify the derivation and presentation. Other crack propagation equations[6,12,13] should be used, if necessary, to account for more general situations. Furthermore, the statistical variability of crack propagation rate, such as Q and b in Eq. (10), has been neglected in the present analysis. This approximation may not be critical, since available test results under random stress history (see Refs. 8-11) indicate small statistical dispersion. Further investigation in this aspect is needed.

Although numerical examples deal with multiple details in one single region, the formulation given by Eqs. (7-9) accounts for the problems of multiple details and multiple regions.

The statistical uncertainty in crack detection is accounted for by the detection probability $F(y)$ that is established from available data.[1-6,14] Hence, there exist certain probability $F^*(y)=1-F(y)$ of missing a crack of size y. If y is large, i.e., large crack is missed during inspection, the structural reliability is low and hence the expected cost of failure is high. Both the probability of structural failure and the expected cost of failure under periodic inspections have been discussed.[1,2]

Occasionally, the NDI detection may reject a detail (i.e., detect a crack at the detail) even if it has extremely small crack (or no crack). This type of uncertainty is ignored in the present analysis because of the lack of background data. It can, however, be taken care of in the present analysis with some modification if the functional form of such an uncertainty is known.

Appendix: Derivation for $p_1^*(i,k)$

After the detected cracks are repaired [Eq. (15)] at T, the probability density, $\bar{f}_{a(T+)}(y_1)$, of the crack size consists of two parts; a) the contribution from unrepaired details $f_{a(T+)}(y_1)$ and b) the contribution from repaired (renewal details $\tilde{f}_{a(T+)}(y_1)$, i.e., $\bar{f}_{a(T+)}(y_1)=f_{a(T+)}(y_1)+\tilde{f}_{a(T+)}(y_1)$. The contribution from unrepaired details, $f_{a(T+)}(y_1)$ follows from Eq. (13) as

$$f_{a(T+)}(y_1)=F^*(y_1)f_{a(0)}(y_2)\,|J(y_1)| \qquad \text{(A1)}$$

in which $y_2=y_1/[1+QTy_1]$. The contribution from repaired details is $\tilde{f}_{a(T+)}(y_1)=p_1(1)f_{a(0)}(y_1)$ where $p_1(1)$ is the probability (or percentage) of repair at T as given by Eqs. (15) and (18).

It is important to note that the integration of $f_{a(T+)}(y_1)$ in Eq. (A1), from 0 to ∞ is exactly $1-p(1)$, that is the percentage of the remaining unrepaired details. Since only the portion of the density function of the crack size, which is contributed by the unrepaired detail is of relevance to the derivation of $p_1^*(i,k)$, only $f_{a(T+)}(y_1)$, Eq. (A1), will be considered in the following deviation.

Owing to the crack propagation, the crack size, $a(2T-)$, for unrepaired details right before the second inspection time is related to $a(T+)$ through the relationship given by Eq. (12) in which $i=2$. Consequently, the portion of density function, $f_{a(2T-)}(y_1)$, contributed by unrepaired details can be obtained from $f_{a(T+)}(y_1)$ given by Eq. (A1) through the transformation of Eq. (12); with the result

$$f_{a(2T-)}(y_1)=F^*(y_2)f_{a(0)}(y_3)\,|J(y_2)|\,|J(y_1)| \qquad \text{(A2)}$$

in which $y_3=y_2/[1+TQy_2]$, $y_2=y_1/[1+TQy_1]$.

The probability of detecting a kth level crack at the second inspection time, $p_1^*(2,k)$, contributed by unrepaired details, is

$$p_1^*(2,k)=\int_{a_{k-1}}^{a_k}F(y_1)f_{a(2T-)}(y_1)\,\mathrm{d}y_1 \qquad \text{(A3)}$$

Substitution of Eq. (A2) into Eq. (A3) yields

$$p_1^*(2,k)=\int_{a_{k-1}}^{a_k}F(y_1)F^*(y_2)f_{a(0)}(y_3)\times|J(y_2)|\,|J(y_1)|\,\mathrm{d}y_1 \qquad \text{(A4)}$$

After the detected cracks have been repaired at the second inspection, the portion of the density funciton $f_{a(2T+)}(y_1)$ of the crack size $a(2T+)$ contributed by unrepaired details follows from Eq. (A2)

$$f_{a(2T+)}(y_1)=F^*(y_1)F^*(y_2)f_{a(0)}(y_3)\,|J(y_2)|\,|J(y_1)| \qquad \text{(A5)}$$

In a similar manner, one can derive the probability of detecting a kth level crack, $p_1^*(i,k)$, during the ith inspection time, contributed by unrepaired details as

$$p_1^*(i,k)=\int_{a_{k-1}}^{a_k}F(y_1)\Big[\prod_{n=2}^{i}F^*(y_n)\Big]f_{a(0)}(y_{i+1})\times\Big[\prod_{n=1}^{i}|J(y_n)|\Big]\mathrm{d}y_1\ ; \qquad i=2,3,\ldots \qquad \text{(A6)}$$

in which

$$y_{n+1}=y_n/[1+TQy_n]=y_1/[1+nTQy_1] \qquad n=1,2,\ldots,i \qquad \text{(A7)}$$

Note that the integration of $f_{a(2T+)}(y_1)$ given by Eq. (A5) from 0 to ∞ represents the percentage of the remaining unrepaired details after the second inspection. It can be shown that the result of integration is smaller than $1-p_1(1)$ due to the repair made at $2T$. Hence, as the number of inspections and repairs (or service time) increases, the percentage of remaining unrepaired details decreases. As can be seen, such a decrease has been accounted for in deriving $p_1^*(i,k)$, Eqs. (A1) and (A6).

References

[1] Yang, J.-N. and Trapp, W. J., "Inspection Frequency Optimization for Aircraft Structures Based on Reliability Analysis," *Journal of Aircraft*, Vol. 12, May 1975, pp. 494-496.

[2] Yang, J.-N. and Trapp, W.J., "Reliability Analysis of Aircraft Structures under Random Loading and Periodic Inspection," *AIAA Journal*, Vol. 12, Dec. 1974, pp. 1623-1630.

[3] Davidson, J. R., "Reliability and Structural Integrity," presented at the 10th Ann. Meeting of the Soc. of Engr. Sci., N.C., 1973.

[4] Davidson, J. R., "Reliability After Inspection," in *Fatigue of Composite Materials*, Amer. Soc. for Testing Materials, ASTM STP 569, 1975, pp. 323-334.

[5] Whittaker, I. C. and Saunders, S. C., "Application of Reliability Analysis to Aircraft Structures Subjected to Fatigue Crack Growth

and Periodic Inspection," Air Force Materials Lab., Wright Patterson AFB, Ohio, AFML-TR-73-92, 1973.

[6]Graham, T. W. and Tetelman, A. S., "The Use of Crack Size Distribution and Crack Detection for Determining the Probability of Fatigue Failure," AIAA Paper 74-394, Las Vegas, Nev., 1974.

[7]Elber, W. and Davidson, J. R., "A Material Selection Method Based on Material Properties and Operating Parameters," NASA TN-D-7221, April 1973.

[8]Paris, P. E., "The Fracture Mechanics Approach to Fatigue," *Fatigue, an Interdisciplinary Approach,* ed. by J. J. Burke, Syracuse Univ. Press, Syracuse, N.Y., 1964.

[9]Smith, S. H., "Fracture Crack Growth Under Axial Narrow and Broad Band Random Loading," *Acoustical Fatigue in Aerospace Structures,* ed. by W. J. Trapp and D. M. Forney, Syracuse Univ. Press, Syracuse, N.Y., 1965.

[10]Smith, S. H., "Random-Loading Fatigue Crack growth Behavior of Some Aluminum and Titanium Alloys," *Structural Fatigue in Aircraft,* Amer. Soc. for Testing and Materials, ASTM STP 404, 1966.

[11]Yang, J.-N., "Statistics of Random Loading Relevant to Fatigue," *Journal of Engr. Mech. Div., ASCE,* Vol. 100, 1974, pp. 469-475.

[12]Gallagher, J. P. and Stanaker, H. D., "Methods for Analyzing Fatigue Crack Growth Rate Behavior Associated with Flight-by-Flight Loading," *Journal of Aircraft,* Vol. 12, Sept. 1975, pp. 699-705.

[13]Porter, T. R., "Method of Analysis and Prediction for Variable Amplitude Fatigue Crack Growth," *Journal of Engineering Fracture Mechanics,* Vol. 4, 1972.

[14]Packman, P. F., et al., "The Applicability of a Fracture Mechanics-Nondestructive Testing Design Criteria," Air Force Materials Lab., Wright Patterson AFB, Ohio, AFML-TR-68-32, 1968.

[15]Tiffany, C. F., "The Design and Development of Fracture Resistant Structures," *Proceedings of the Colloquium on Structural Reliability,* Carnegie-Mellon Univ., 1972, pp. 210-215.

[16]Crichlow, W. J., "On Fatigue Analysis and Testing for the Design of the Airframe," in *Fatigue Life Prediction for Aircraft Structures and Materials,* AGARD-LS-62, 1973, pp. 6.1-6.36.

[17]Butler, J. P., "The Material Selection and Structural Development Process for Aircraft Structural Integrity Under Fatigue Conditions," *Proceedings of the Air Force Conference on Fatigue and Fracture of Aircraft Structures and Materials,* Air Force Flight Dynamics Lab., Wright Patterson AFB, Ohio, USAFFDL-TR-70-144, 1970, pp. 17-44.

[18]Hahn, G. J. and Shapiro, S. S., *Statistical Models in Engineering,* Wiley, N. Y., 1967.